Mathematics is a Game Played with Symbols

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Abstract: Mathematics—at least formal mathematics—is a type of game. This game is played with symbolic shapes, and uses rules that maintain systematic relationships with a reference class of situations. Generally, formal expressions can be treated as objects that move in constrained ways through discrete states. The form of expressions provides one type of interface through which to engage mathematical meanings. We suggest here that this interface is largely outdated, relying on modes of interaction which have changed little in many decades, and that it is at least plausible that some of the anxiety experienced by many students when they encounter algebra results from the primitive nature of the interface, rather than from the game of algebra itself. We present the results of one experiment conducted in an urban middle school exploring learning in a highly structured mathematics lesson using the *Pushing Symbols* framework.

Introduction

Educational algebra games often take as an assumption that algebra is intrinsically boring. Algebraic content, in these games, derives value from being embedded in some interesting game context (e.g., *Ko's Journey; Dimension M*); indeed, it is sometimes suggested that interesting games that are intrinsically about algebra are impossible (Devlin, 2010). Along similar lines, developmental psychologists have suggested that while number and space are intrinsically interesting domains of reasoning, algebraic equations are not (Carey, 2009). In contrast, mathematicians and philosophers who describe symbolic notational systems often characterize symbol systems themselves as interesting games already--games involving the manipulation of physical objects (e.g., Haugeland, 1981). In this game, variables, operation signs, and constant symbols are pieces—physical objects—moved according to set rules. The purpose of algebraic mathematics is then to explore games that can be played within particular rule sets, and to infer properties of those games. On this account, algebras and algebraic propositions bear a kinship to physical puzzles such as Rubik's cubes: they depict objects that undergo particular transformations and exist in certain relations.

When algebra is taught, the idea of algebra as a game over physical tokens is rarely presented, and indeed physicalizing symbols is often seen as an error (e.g., Nogueira de Lima & Tall, 2007). Instead, algebraic systems are characterized by textbooks as linguistic systems, with explicit rules that must be memorized or comprehended. Because these rules appear arbitrary, they are often introduced through analogy from real-world examples, or even from the observation of patterns in particular numerical problems. The approach taken here is to apply technology—particularly touchscreen interfaces—to align the content of algebraic transformations with core cognitive and perceptual systems by making the objects of the symbolic metaphor into literal objects, which children can touch and move, and which respond in natural, object-like ways. Students can then engage in goal-directed play in a system that presents 'challenges' based on strategies and concepts involved in expression manipulation. We predicted that this system would help students engage in algebraic content more naturally, and that this interaction might increase engagement with mathematical symbol systems.

Experiment

98 sixth and seventh graders participated in a randomized-design experiment involving the pushing symbols approach. The experiment took place over 4 sessions in the students' regular classrooms. The first and last sessions were limited to a pretest and 1-month retention test; the central two days involved a lesson in the combination of like terms. There were two like terms lessons; each student received each lesson in counterbalanced order. One lesson was intended as an object-focused and game-based presentation of like terms (the *Pushing Symbols* condition); the other was intended to serve as a 'best practices' control. In each case, the lesson consisted of a short (about 5-10 minutes) lesson using the whiteboard, a physical tile activity (5 minutes), and practice using an iPad-based activity (about 30 minutes). For the Pushing Symbols group the iPad activity was a modified form of the *Algebra Touch* video game. For the control group, the activity involved using a digital pen to enter answers in a format much like a paper-and-pencil test. Each problem was followed by a worked solution. All tests consisted of 18 paper-and-pencil like terms problems.

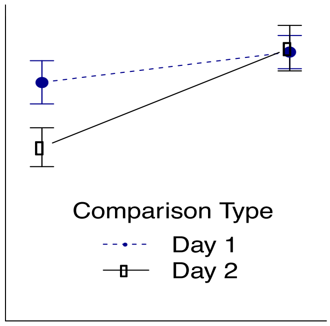

Figure 1: Mean accuracy (left), and engagement (right). Errors are standard errors.

Results

For this study, we investigated one learning outcome (structure learning) and a general measure of engagement for each of the tools. To evaluate learning differences between groups at different testing time points, we used t-tests to examine the proportion of problems solved without structure errors (see Figure 1). At pretest, there were no group differences in achievement, $t(95)=-0.78$, $p=0.44$. After the first day of instruction, students in both groups made significant gains in their understanding of simplifying expressions; however, no significant differences between those who received the Pushing Symbols intervention or the control intervention were found, $t(91)=0.17$, $p=0.86$. After the second day of instruction, significant differences emerged between conditions, $t(92)=-2.85$, $p<0.01$), suggesting an order effect and benefit of receiving the Pushing Symbols intervention first. Retention assessments 1 month later demonstrated that students in both groups retained a level of mastery for simplifying expressions.

Our second prediction was that engagement in the mathematical lesson would be higher when the algebraic interface engagement overall was quite high. Mean engagement was .75 (*SE*=1.1). An ANOVA of engagement against condition indicated higher engagement among students in the PS than in the control condition, $F(1, 86)=4.5$, $p<0.05$.

Discussion and Future Plans

A dynamic interface that allows object-like interactions with the structures depicted by mathematical symbols can be situated in a traditional lesson, and doing so can positively impact achievement and engagement. These results have implications for educators and researchers studying mathematical cognition. Examinations of algebra learning have largely been rooted, necessarily, in counterintuitive notation systems whose mastery involves explicit memorization of rules with minimal perceptual support. These results provide a preliminary demonstration that basic algebra lessons that align axiomatic algebraic content, goals, and perceptual and motor activity can yield substantial learning at least comparable to that of worked examples in the context of conceptually-oriented lessons. They call for careful investigation of learning outcomes in symbolic algebras that are intrinsically physical.

References

Carey, S. (2009). *The origin of concepts*. New York: Oxford University Press.

Devlin, K. (2010). *Mathematics Education for a New Era: Video Games as a Medium for Learning.* A K Peters/CRC Press: Boca Raton, FL.

Haugeland, J. (1981). Semantic engines: An introduction to mind design. *Mind Design*. J. Haugeland (Ed.), MIT Press: Cambridge, MA.

Nogueira de Lima, R. & Tall, D. (2008). Procedural embodiment and magic in linear equations. *Educational Studies in Mathematics*, *67* (1) 3-18.